THE QUANTIZATION OF GRAVITY. DYNAMIC APPROACH.

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Abstract.

On the basis of dynamic quantization method we build in this paper a new mathematically correct quantization scheme of gravity. In the frame of this scheme we develop a canonical formalism in tetrad-connection variables in 4-D theory of pure gravity. In this formalism the regularized quantized fields corresponding to the classical tetrad and connection fields are constructed. It is shown, that the regularized fields satisfy to general covariant equations of motion, which have the classical form. In order to solve these equations the iterative procedure is offered.

1. Introduction.

In this paper we state the mathematically correct quantization scheme of gravity in 4-dimensional space-time. The basis of the scheme is the dynamic quantization method. The dynamic quantization method was already successfully applied to gravity interacting with Dirac field in 2+1-dimensional space-time. The regularization conserving general covariance of the theory was carried out and perturbation theory (PT) was constructed [1,2]. The idea of the dynamic quantization method is based on the Dirac theory of quantization of systems with constraints and in particular of general covariant systems.

Describe briefly the dynamic quantization method. As is known [3], the hamiltonian in general covariant theories is arbitrary linear combination of the first class constraints χ_α. If $|\mathcal{M}\rangle$ is any physical state, then $\chi_\alpha|\mathcal{M}\rangle = 0$. Let $|\mathcal{N}\rangle = a^+|\mathcal{M}\rangle$ be other physical state and a^+ be some operator of creation type. As the hamiltonian annuls all physical states, then can assume that $[\chi_\alpha, a^+] = 0$. There is an infinite number of creation and annihilation type operators $a_N^+,\ a_N$ which transfer one physical states in others and exhausting all local physical degrees of freedom of the system. The all operators $\{a_N, a_N^+\}$ are conserved, since they commute with the hamiltonian. This imply that any set of pairs of the operators $\{a_N, a_N^+\}''$ can be considered as a set of the second class constraints in the Dirac sense [3]. This fact gives the following possibility for regularization in considered theory: the regularization of the system is made by imposing of an infinite set of second class constraints

$$a_N = 0, \qquad a_N^+ = 0\,, \qquad |N| > N_0 \tag{1.1}$$

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Thus, in the theory remains only final number of degrees of freedom, corresponding to the operators a_N^+ and a_N with $|N| < N_0$. The final set of the remained operators $\{a_N,\, a_N^+\}'$ corresponds to the set of physical states which describe enough completely the investigated system. As a result a Poisson bracket is replaced by the corresponding Dirac bracket. It is critically important that under such regularization the equations of motion do not change its classical form (see section 4). This fact imply that the general covariance in regularized theory is conserved. Moreover, retaining a "small" number of physical degrees of freedom and states in the theory, we obtain a new possibility of development of PT in the number of the remained physical degrees of freedom.

Not concerning the complete review of other directions of canonical quantization of gravity, we pay attention to results of one of roughly developed schools, which can be presented by works [4] (see the references there). Within the framework of developed by the authors of [4] technic the principal possibility of canonical non-perturbative quantization of gravity is elaborated: the physical states of theory are described, they form the normalized space; in this space the construction of the linear operators, to the number of which the first class constraints or hamiltonian belong, is carried out; the problem of construction of physical states annuling the hamiltonian is solved; the commutational relations between the first class constraints do not contain undesirable Schwinger terms. In our opinion these results by an indirect way corroborate our method, causing the same general results.

The method of dynamic quantization is constructed on principles of canonical quantum theory. Therefore for its application it is necessary previously to develope the apparatus of classical hamiltonian mechanics. This problem is solved in Section 2. In Section 3 the formal construction of a quantum theory is performed. Since in theory of gravity the formal quantization is mathematiucally not correct, the account in this Section has heuristic character. In Section 4 the successive logically and mathematiucally correct quantum theory of gravity is built. In Section 5 the perturbation theory is developed.

2. Canonical formalism.

We shall consider the first order vierbein action of pure gravity in 4-D space-time:

$$A = -\frac{1}{8\kappa^2}\int d^4x\, \varepsilon^{\mu\nu\lambda\rho}\varepsilon_{abcd}e^c_\lambda e^d_\rho\, R^{ab}_{\mu\nu} = \int dx^0\, \mathcal{L},$$

$$R^{ab}_{\mu\nu} = \partial_\mu\omega^{ab}_\nu - \partial_\nu\omega^{ab}_\mu + \omega^a_{\mu g}\omega^{gb}_\nu - \omega^a_{\nu g}\omega^{gb}_\mu \qquad (2.1)$$

Here e^a_μ are tetrads, so that $g_{\mu\nu} = \eta_{ab}\, e^a_\mu e^b_\nu$ is a metric tensor in local coordinates $x^\mu = (x^0,\, x^i)$, $\mu, \nu, \ldots = 0, 1, 2, 3$ are coordinate indexes. The $a, b, c, \ldots = 0, 1, 2, 3$

are local Lorentz indexes, $\eta_{ab} = diag(1,-1,-1,-1)$ is the Lorentz metrics and $\omega_\mu^{ab} = -\omega_\mu^{ba}$ is the connection in orthonormal basis e_μ^a and the covariant derivative of the vector $\xi^a = e_\mu^a \xi^\mu$ is of the form

$$\nabla_\mu \xi^a = \partial_\mu \xi^a + \omega_\mu^{ab} \xi_b$$

Further, let $\varepsilon^{\mu\nu\lambda\rho}$ ($\varepsilon^{0123} = 1$) and ε_{abcd} ($\varepsilon_{0123} = 1$) be completely antisymmetric pseudotensors, referred to coordinate and Lorentz basises, accordingly.

Now begin the construction of canonical formalism for action (2.1) in the form convenient for us. This problem was repeatedly considered in canonically-conjugated variables ω_i^{ab} and

$$\mathcal{P}^i_{ab} \equiv \frac{\partial \mathcal{L}}{\partial \dot{\omega}_i^{ab}} = -(2\kappa^2)^{-1}\, \varepsilon_{ijk}\, \varepsilon_{abcd}\, e_j^c e_k^d \tag{2.2}$$

(See, for example, [5]). The point on top means derivative in time $t = x^0$. However, since there are the second class constraints in the system (2.1) the Poisson brackets in variables $\{\omega_i^{ab},\, \mathcal{P}^i_{ab}\}$ appear rather complicated. Moreover, the equations of motion in these variables are enough bulky. On the other hand, in variables $\{\omega_i^{ab},\, e_j^c\,\}$ as equations of motion, so the constraints look as much as possible simply. This circumstance is extremely important for us, since in the dynamic quantization methodof the equations of motion play a main role in principal calculations. The tetrad-connection variables are prefereble also because just in these variables the property of supersymmetry in supergravity is formulated. Thus the development of hamiltonian formalism in variables $\{\omega_i^{ab},\, e_j^c\,\}$ having direct physical sense seems for us more rational.

By definition

$$\mathcal{L} = \frac{\partial \mathcal{L}}{\partial \dot{q}}\, \dot{q} - H$$

where q are some generalized coordinates. In our case the hamiltonian is of the form

$$H = \int d^3x\, \{-\frac{1}{2}\omega_0^{ab}\chi_{ab} + \frac{1}{2\kappa^2} e_0^c \phi_c\,\}\ ,$$

$$\chi_{ab} = \frac{1}{\kappa^2}\varepsilon_{abcd}\varepsilon_{ijk} e_i^c \nabla_j e_k^d\ , \qquad \phi_c = \frac{1}{2}\varepsilon_{abcd}\varepsilon_{ijk} e_k^d R_{ij}^{ab} \tag{2.3}$$

Here $\nabla_\mu e_\nu^c = \partial_\mu e_\nu^c + \omega_\mu^{cb} e_{b\nu},\ \omega_0^{ab}$ and e_0^c are arbitrary functions, playing a role of Lagrange multipliers. Let us express the action (2.1) in the form

$$A = -\int d^4x\, \frac{1}{4\kappa^2}\, \varepsilon_{abcd}\varepsilon_{ijk} e_j^c e_k^d \dot{\omega}_j^{ab} - \int dt\, H \tag{2.4}$$

From the condition $\delta A = 0$ we find two equations relatively to $\dot{\omega}_i^{ab}$ and $\dot{e}_j^c$:

$$\frac{1}{2\kappa^2}\, \varepsilon_{abcd}\varepsilon_{ijk} e_k^d \dot{\omega}_i^{ab} + \frac{\partial H}{\partial e_j^c} = 0\ , \tag{2.5}$$

$$\frac{1}{2\kappa^2}\,\varepsilon_{abcd}\varepsilon_{ijk}\dot{e}^c_j e^d_k - \frac{\partial H}{\partial \omega^{ab}_i} = 0 \tag{2.6}$$

Eq.(2.6) has the solution only under additional conditions

$$\lambda^{ij} = (g^{ik}\varepsilon_{jlm} + g^{jk}\varepsilon_{ilm}\,)\,e_{ak}\nabla_l e^a_m = 0\ , \tag{2.7}$$

where $g^{ik}g_{kj} = \delta^i_j$. It is necessary to consider Eq. (2.7) as the second class constraints. It is seen from the fact that the six equations (2.7) in each point x reduce the number of independent variables $\omega^{ab}_i(x)$ from eighteen up to twelve. The number of independent variables $e^a_i(x)$ is also equal to twelve. From Eq.(2.6) under conditions (2.7) we find:

$$\nabla_0 e^a_i = \nabla_i e^a_0 \tag{2.8}$$

Besides the constraints $\chi_{ab} \approx 0$ and Eq.(2.7) give

$$\nabla_i e^a_j - \nabla_j e^a_i = 0\ (mod\ \chi_{ab}) \tag{2.9}$$

As is known, the connection is expressed unambiguously from Eq. (2.8) and (2.9) through tetrads and its derivatives. Eq.(2.5) determines the quantity $\dot{\omega}^{ab}_i$ up to the term $\varepsilon_{jkl}e^a_k e^b_l s_{ij},\ s_{ij} = s_{ji}$. It is possible to find equations of motion with requirement of conservation for constraints (2.7). Write out the equations of motion for the connection:

$$R^{ab}_{0i} = \frac{1}{2}\varepsilon_c{}^{dab}\varepsilon_{jkl}\tilde{e}_{0d}e_{if}e_{0g}e^{(f}_l R^{c)g}_{jk} -$$
$$\frac{1}{2}\,(\tilde{e}^{[a}_0 e^{b]}_l e^c_i + \frac{1}{2}\,e^{[a}_l e^{b]}_i \tilde{e}^c_0\,)\cdot\varepsilon_{cdfg}\varepsilon_{jkl}e^d_0 R^{fg}_{jk}\ \ Mod(\chi_{ab},\,\phi_c\,) \tag{2.10}$$

Here and further $\tilde{e}_{0a} = -g^{-1}\varepsilon_{abcd}e^b_1 e^c_2 e^d_3,\ g = det\,g_{ij}$ and $(a\,b)$ (or $[a\,b]$) means symmetrization (antisymmetrization) relative to the pair of indexes in the brackets.

Note that Eq. (2.10) and the constraints $\phi_c = 0$ are contained in equations

$$\varepsilon^{\mu\nu\lambda\rho}\varepsilon_{abcd}e^c_\lambda R^{ab}_{\mu\nu} = 0 \tag{2.11}$$

According to definition of Poisson bracket (PB) the equations of motion (2.8), (2.10) can be written in the form

$$\dot{A} = [A,\,H\,]$$

Here $[\ldots,\ldots]$ designates Poisson bracket of the quantities A and H and A is any function of dynamic variables.

Eqs (2.5) and (2.6) and also the conditions $[e^a_i(x),\,\lambda^{jk}(y)\,] = 0$ determine unambiguously the following:

$$[e^a_i(x),\,e^b_j(y)] = 0\ , \tag{2.12}$$

$$[\omega^{bc}_j(x),\,e^a_i(y)] = \delta(x-y)\,\kappa^2\,\{\,2\,\tilde{e}^{[b}_0\,e^{c]}_i\,e^a_j +$$

$$+e_i^{[b} e_j^{c]} \, \tilde{e}_0^a + e_j^{[b} \, \tilde{e}_0^{c]} \, e_i^a \}(x), \tag{2.13}$$

To find connection-connection Poisson brackets more long calculations are required. Since further the connection-connection Poisson brackets do not used obviously, they are not here written out.

For completion of the description of hamiltonian formalism we shall show, that under conditions (2.7) the quantities χ_{ab} and ϕ_c are the first class constraints.

From (2.3), (2.8) and (2.10) follows, that quantities $\chi_{ab}(x)$ generate the local Lorentz transformations in point x . In particular

$$[\chi_{ab}(x), \, e_i^c(y) \,] = -\delta(x-y)(\, \delta_a^c e_{bi} - \delta_b^c e_{ai} \,)(x),$$

$$[\chi_{ab}(x), \, \omega_i^{cd}(y) \,] = -2 \, \partial_i \delta(x-y) \, \delta_a^{[c} \delta_b^{d]} -$$

$$2 \, \delta(x-y) \, \{ \delta_a^{[c} \omega_{b \, i}^{\;\; d]} + \delta_a^{[d} \omega_i^{\; c]b} \, \} \, (x)$$

Any Lorentz-tensor quantity has the Poisson bracket with the quantity χ_{ab} similar to the written out. Therefore the equation of motion for χ_{ab} is of the form

$$\dot{\chi}^{ab} = 2 \, \omega_{0 \;\; c}^{[a} \chi^{b]c} - \frac{1}{\kappa^2} \, e_0^{[a} \phi^{b]}$$

Thus

$$\partial_\mu \chi^{ab} = 0 \; (mod \; \chi_{ab}, \phi_c \,) \tag{2.14}$$

Differentiating in space-time coordinates the curvature tensor (2.1), we find

$$\nabla_\lambda R_{\mu\nu}^{ab} + \nabla_\mu R_{\nu\lambda}^{ab} + \nabla_\nu R_{\lambda\mu}^{ab} = 0 \; , \tag{2.15a}$$

where

$$\nabla_\lambda R_{\mu\nu}^{ab} = \partial_\lambda R_{\mu\nu}^{ab} + \omega_{\lambda c}^a R_{\mu\nu}^{cb} + \omega_{\lambda c}^b R_{\mu\nu}^{ac}$$

Further, differentiating Eqs (2.8) and (2.9) and using (2.14) we obtain

$$R_{\mu\nu}^{ab} e_{b\lambda} + R_{\nu\lambda}^{ab} e_{b\mu} + R_{\lambda\mu}^{ab} e_{b\nu} = 0 \; (mod \; \chi_{ab}, \; \phi_c \,) \tag{2.15b}$$

The relations (2.15) mean that Bianchi identities in canonical formalism are valid.

We state now, that

$$\dot{\phi}_c = 0 \; (Mod \; \chi_{ab}, \; \phi_c \,) \tag{2.16}$$

The equations (2.14) and (2.16) mean that the constraints $\chi_{ab}, \, \phi_c$ are the first class constraints.

Let us show the correctness of Eq. (2.16). From definition of quantities ϕ_a we have

$$\nabla_0 \phi_a \sim \varepsilon_{abcd} \varepsilon_{ijk} \, (\, \nabla_0 e_i^b \cdot R_{jk}^{cd} + e_i^b \nabla_0 R_{jk}^{cd} \,) =$$

$$\varepsilon_{abcd} \varepsilon_{ijk} \, [\nabla_i e_0^b \cdot R_{jk}^{cd} - e_i^b \, (\, \nabla_j R_{k0}^{cd} + \nabla_k R_{0j}^{cd} \,) \,] \tag{2.17}$$

The last equality is based on Eq.(2.8) and identity (2.15a). Using (2.9) and (2.15), we represent the right hand side of (2.17) up to terms proportional to the constraints χ_{ab} or ϕ_c as follows:

$$\nabla_i \left[\varepsilon_{abcd} \varepsilon_{ijk} \left(e_0^b R_{jk}^{cd} - 2\, e_j^b R_{0k}^{cd} \right) \right] \, (mod \; \chi_{ab}, \, \phi_c \,) \tag{2.18}$$

The quantity in square brackets in (2.18) can be written in the form $\frac{\delta \mathcal{L}}{\delta e_i^a}$ which is equal to zero due to equations of motion. Thus equality (2.16) is proven.

3. Formal quantization.

Passing from the classical mechanics to quantum one we must replace classical Poisson brackets by quantum coommutation relations. It is usually assumed that quantum Poisson bracket for fundamental variables differ from classical one only by a multiplier $i(\hbar = 1)$ which in our case stands in right hand side of Eqs (2.12) and (2.13). The Heisenberg equations $i\,\dot{A} = [\,A, \, H\,]$ for variables e_i^a and ω_i^{ab} save its classical form up to operators ordering.

From commutators (2.12), (2.13) it follows, that the set of variables $\{e_i^a(x)\}$ is a complete set of mutually commuting variables. The possible values of these variables satisfy to the conditions

$$-\infty < e_i^a(x) < +\infty$$

We shall write out the formula for operator of connection. As in this Section a formal quantum theory is considered, the problem of correct ordering of operators is neglected here. Using classical PB (2.12) and Eq. (2.7) we find:

$$\omega_j^{bc}(x) = \kappa^2 \left\{ 2\tilde{e}_0^{[b}\, e_i^{c]}\, e_j^a + e_i^{[b} e_j^{c]}\, \tilde{e}_0^a + e_j^{[b}\, \tilde{e}_0^{c]}\, e_i^a \right\} \pi_a^i(x) +$$

$$+ (2g)^{-1} \varepsilon_{klm} \varepsilon_{inp} e_n^b e_p^c \,\partial_l e_m^d \cdot \{\, g_{ij} e_{dk} - g_{jk} e_{di} - g_{ik} e_{dj} \,\} \tag{3.1}$$

Here the field $\pi_a^i(x)$ is defined by the formulae

$$[\,\pi_a^i(x), \, \pi_b^j(y)\,] = 0, \qquad [\,\pi_a^i(x), \, e_j^b(y)\,] = i\, \delta_j^i \delta_a^b \delta\,(x - y) \tag{3.2}$$

Pass now to the problem of finding of the conserved operators, which exshaust the all local physical degrees of freedom. The conserved operators in general covariant theories can be named also as gauge-invariant operators. The problem of construction of gauge-invariant operators is resolved much more convenient by an axiomatic approach similar to the case of 3-space-time theory of gravity [2].

Let us introduce the following natural assumptions or axioms about the structure of the physical space of states F.

Axiom 1. *All states of the theory, having the physical sense, are obtained from the ground state* $|\,0\,\rangle$ *with the help of the creation operators* a^+_N :

$$|\,n_1,\,N_1;\ldots;\,n_s,\,N_s\,\rangle = (n_1!\cdot\ldots\cdot n_s!\,)^{-\frac{1}{2}}\cdot(a^+_{N_1})^{n_1}\cdot\ldots\cdot(a^+_{N_s})^{n_s}\,|\,0\,\rangle\ ,$$

$$a_N\,|\,0\,\rangle = 0 \tag{3.3}$$

The states (3.3) form orthonormal basis of the physical states space F *of the theory.*

The numbers $n_1,\ldots,n_s$ take the natural valuess and are called by occupation numbers.

Axiom 2. *The states (3.3) satisfy to the conditions:*

$$\chi_{ab}(x)\,|\,n_1,\,N_1;\ldots;\,n_s,\,N_s\,\rangle = 0,$$

$$\phi_a(x)\,|\,n_1,\,N_1;\ldots;\,n_s,\,N_s\,\rangle = 0 \tag{3.4}$$

Axiom 3. *The state* $e^a_i(x)\,|\,n_1,N_1;\ldots;n_s,N_s\rangle$ *contains a superposition of* **all** *states for which one of occupation number differs per unit from occupation number of the state (3.3) and the rest coincide with corresponding occupation numbers of the state (3.3).*

The operators a^+_N and their Hermithian conjugated a_N have usual commutational properties:

$$[\,a_N,\,a_M\,] = 0, \qquad [a_N,\,a^+_M\,] = \delta_{NM} \tag{3.5}$$

For the case of the compact spaces one can consider, that index N numbering creation and annihilation operators belongs to the discret finite dimensional lattice. In the space of indexes N the norm can be easily introduced.

From Eqs. (3.3) and (3.4) it follows, that $[H,\,a^+_N\,]\approx 0,\qquad [H,\,a_N\,]\approx 0$. We shall accept more strong conditions:

$$[H,\,a^+_N\,] = 0, \qquad [H,\,a_N\,] = 0\ , \tag{3.6}$$

where the operator H is given by (2.3).

Thus interesting for us gauge-invariant operators are formally indicated. This is the set of the annihilation and creation operators $\{a_N,\,a^+_N\}$ exshausting the all local physical degrees of freedom of the system. We pay attention on the fact that commutational relations (CR) (3.6) are in agreement with general covariance of the theory. In other theories there is no the set of the operators with properties (3.6) exshausting physical degrees of freedom of the system.

Give some consequences from Axioms 1-3.

Let $|\,N\,\rangle = a_N^+\,|\,0\,\rangle$. From Axiom 3 follows, that

$$e_i^a(x)\,|\,N\,\rangle = \frac{1}{\sqrt{2}} e_{Ni}^a(x)\,|\,0\,\rangle + |\,N;\, e_i^a(x)\,\rangle,$$
$$\langle\,0\,|\,N;\, e_i^a(x)\,\rangle = 0, \tag{3.7}$$

The fields $e_{Ni}^a(x)$ are linearly independent and by definition

$$[\,e_{Ni}^a(x),\, a_M\,] = 0, \qquad [\,e_{Ni}^a(x),\, a_M^+\,] = 0 \tag{3.8}$$

Since the field $e_i^a(x)$ is Hermithian there is the following expansion:

$$e_i^a(x) = \frac{1}{\sqrt{2}} \sum_N (a_N\, e_{Ni}^a(x) + a_N^+\, \bar{e}_{Ni}^a(x)\,) + \tilde{e}_i^a(x), \tag{3.9}$$

The field $\tilde{e}_i^a(x)$ does not contain the operators a_N and a_N^+ in the first power, but contains the contribution of a zero power which we shall designate by $e_i^{a(0)}(x)$.

The information about configuration of the fields $e_{Ni}^a(x)$ can be received by study the matrix elements of some invariant operators relative to the states (3.3). For example, consider the quantity $V = \int d^3x\,\sqrt{-g}$ which is invariant relative to general coordinate transformations in 3-D space. As it is known, the expression in the right hand side of Eq. (2.8) gives the change of tetrads under the infinitesimal transformation. On the other hand, the right hand side of Eq.(2.8) is equal to PB of tetrads and hamiltonian. Since the hamiltonian annulates the physical states, there is the equality

$$\langle\,0\,|\,(\int d^3x\,\sqrt{-g}\,)\,|\,N\,\rangle = \langle\,0\,|\; e^{-i\varepsilon H}\,(\int d^3x\,\sqrt{-g}\,)\,e^{i\varepsilon H}\,|\,N\,\rangle$$

From here we obtain the equation

$$\langle\,0|\;:\{\int d^3x\,\sqrt{-g}\,g^{ij}\,(\,e_j^a\nabla_i\xi_a + \nabla_i\xi_a\cdot e_j^a\,)\,\}\;:\;|\,N\,\rangle = 0, \tag{3.10}$$

which is true for any field $\xi^a(x)$.

To advance further, we shall assume, that the Heisenberg tetrads and connection fields can be expressed as formal series in operators a_N and a_N^+ . For the tetrad fields the beginning of this expansion is given by Eq. (3.9). Then we have the expansion for the connection operator accordin to Eq. (3.1). Let designate by $\omega_i^{ab(0)}(x)$ the zero power term in operators a_N and a_N^+ . This term can be expressed through the field $e_i^{a(0)}(x)$ with the help of Eqs. (2.8), (2.9).

Now from equality (3.10) the following conditions for fields e_{Ni}^a are obtained

$$\nabla_i^{(0)}\,(\sqrt{-g^{(0)}}\,g^{ij(0)}e_{Nj}^a\,) = 0 \tag{3.11}$$

The top index (0) means that the all operators and fields under this index depend only on zero approximation of the tetrad and connection fields $e_i^{a(0)}$ and $\omega_i^{ab(0)}$.

Designate by $e_i^{a(s)}(x)$ and $\omega_i^{ab(s)}(x)$, $s=0,1,\dots$ the contributions to tetrad and connection fields of the power s relative to the creation and annihilation operators, so that

$$e_i^a(x)=\sum_{s=0}^{\infty} e_i^{a(s)}(x)\,,$$

$$e_i^{a(s)}(x)=\sum_{N_1\dots N_s} a_{N_1}\dots a_{N_s}\, e^a_{N_1\dots N_s\,i}(x)+$$

$$\sum_{M_1}\ \sum_{N_1\dots N_{s-1}} a^+_{M_1}\, a_{N_1}\dots a_{N_{s-1}}\, e^a_{M_1;N_1\dots N_{s-1}\,i}(x)+\dots \tag{3.12}$$

Here the creation and annihilation operators are normally ordered. The similar formulae take place for the connection field. All information about evolution in time of the system is contained in the fields $e^a_{N_1\dots N_s\,i}(x)$, $\omega^{ab}_{N_1\dots N_s\,i}(x)$ and so on , $s=0,1,\dots$. We shall designate by $\Phi_{\mathcal{N}}(x)$ the totality of these fields.

Denote the group consisting of elements $\{S^a_b(x)\}$ by G . To each element of group G corresponds the transformation of the fields of tetrads and connections:

$$e_i'^a(x)=S^a_b(x)e^b_i(x),$$

$$\omega_i'^{ab}(x)=S^a_c(x)S^b_d(x)\omega^{cd}_i(x)+S^a_c(x)\eta^{cd}\partial_i S^b_d(x)$$

The operator

$$\chi(\omega_0)=\frac{1}{2}\int d^3x\,\omega_0^{ab}\chi_{ab},\qquad \omega_0^{ab}\longrightarrow 0$$

is the right invariant vector field on the group G transfering the point $S^a_b(x)$ in the infinitely close point $S^a_b(x)-\omega^a_{0c}(x)\,S^c_b(x)$. The vector field on the group $\mathcal{G}$, corresponding to the operator

$$\phi(e_0)=\frac{1}{2\,\kappa^2}\int d^3x\,e_0^c\phi_c,\qquad e_0^c\longrightarrow 0,$$

generates the following shift of tetrad fields (see (2.3) and (2.8)):

$$e^a_i\longrightarrow e^a_i+\nabla_i e^a_0$$

Now it is easy to express the field $\pi^i_a(x)$, conjugated with the tetrad field $e^a_i(x)$ (see (3.2)) in the first approximation relative to the operators a^+_N and a_N . For thees it is necessary to complement the set of fields $e^a_{Ni}(x)$ in (3.9) up to complete set of orthonormal fields $\{e^a_{Ni}(x)\}$. In consequence the following formulae take place:

$$\int d^3x\,\sqrt{-g^{(0)}}\,\bar e^a_{Mi}g^{ij(0)}e_{Naj}=\kappa^2q^2_M\,\eta_{MN}, \tag{3.13}$$

where $\eta_{MN}=0$ if $M\neq N$, $\eta_{NN}=1$ or -1 for the space- or time-like field e^a_{Ni} , accordingly . Here q^2_M is the normalising multiplier having dimension of

lengths. Except the condition of orthonormality (3.13) there is also the condition of completeness:

$$\sum_N \kappa^{-2} q_N^{-2}\, \eta_{NN}\, e^a_{Ni}(x) \bar{e}^b_{Nj}(x) =$$

$$\frac{1}{\sqrt{-g^{(0)}}}\, g^{(0)}_{ij} \eta^{ab} \delta(x-y) - \nabla^{(0)}_i(x) \nabla^{(0)}_j(y) D^{ab(0)}(x,y),$$

$$-\nabla^{(0)}_i \sqrt{-g^{(0)}}\, g^{ij(0)} \nabla^{(0)}_j D^{ab(0)}(x,y) = \eta^{ab} \delta(x-y) \tag{3.14}$$

The set of operators a^+_N and a_N also must be complemented so to have instead CR (3.5) the CR:

$$[a_M,\, a^+_N\,] = \eta_{MN} \tag{3.15}$$

Taking into account formulas (3.9), (3.14) and (3.15), we obtain the following representation for the operator π^i_a :

$$\pi^i_a(x) = i\, (\, \sqrt{-g^{(0)}}\, g^{ij(0)}\,)(x)\, \{\, \frac{1}{\sqrt{2}\, \kappa^2} \sum_N q_N^{-2}\, (\, a_N e_{Naj}(x) - a^+_N \bar{e}_{Naj}(x)\,) +$$

$$\nabla^{(0)}_j \int d^3 z\, D_a{}^{c(0)}(x,z) \frac{\delta}{\delta \xi^c(z)} \} \tag{3.16}$$

The second term in (3.16) is vector field on the group $\mathcal{G}$, so that

$$[\int d^3 z\, e^c_0(z) \frac{\delta}{\delta \xi^c(z)}\,,\; e^a_i(x)\,] = \nabla_i e^a_0(x) \tag{3.17}$$

From formulas (3.1), (3.9) and (3.14) - (3.17) it is seen that
1) in the first approximation relative to operators a^+_N and a_N the fields of tetrads and connections satisfy to quantum PB (2.12) and (2.13);
2) the fields of connections also, just like the fields of tetrads, in the first order relative to the operators a^+_N and a_N contain all these operators.

It is clear from the end of this Section (beginning with (3.13)) that the fields of tetrad and connection have more degrees of freedom, than it is necessary from kinematic considerations. Really, except the degrees of freedom, corresponding to the gauge group $\mathcal{G}$ and contained in the fields $\Phi_{\mathcal{N}}(x)$, there is the overfuled system of the degrees of freedom, contained in the set of all creation and annihilation operators . However, this does not prevent further advancement by the following reasons: under regularization almost all the creation and annihilation operators are eliminated and in the theory only minimal their number remains. Thus under regularization the gauge group remains intact.

Now we have all necessary means for regularization of the theory.

4. Regularization.

The description in previous Section carried the formal character, since the divergences were not taken into account . In this Section the regularization of the theory is carried out. Thereby the mathematical sense is imparted to all used operators and equations.

The importance of CR (3.5) and (3.6) is that every set of pairs of the operators a_N, a_N^+ can be considered as a set of the second class constraints in the Dirac sense [3]. It enables the following realization of regularization.

Select a finite set of pairs of annihilation and creation operators $\{a_N, a_N^+\}'$ and numerate them in such a manner that $|N| < N_0$. The operators from the set $\{a_N, a_N^+\}'$ satisfy CR (3.5). Since the physical information is contained in wave functions $e^a_{Ni}(x)$, this choice is actually defined by the choice of a set of linearly independent wave functions $\{e^a_{Ni}(x)\}'$, corresponding to the set of operators $\{a_N, a_N^+\}'$. The choice of functions in the set $\{e^a_{Ni}(x)\}'$ is determined by physical conditions of a problem. For example, if x-space is torus then as the wave functions of this set the periodical waves with wave numbers limitted by modulus can be taken. Such choice of the set $\{ e^a_{Ni}(x) \}'$ corresponds to the problem of gravitational waves.

The regularization of the theory consists in the following: the all infinite number of pairs of the annihilation and creation operators at $|N| > N_0$, i.e. except chosen, we believe equal to zero:

$$a_N = 0, \qquad a_N^+ = 0, \qquad |N| > N_0 \tag{4.1}$$

Thereby the infinite set of the second class constraints is imposet. Now we must replace CR (2.12), (2.13) et al. to corresponding Dirac CR and investigate the regularized equations of motion.

We shall prove the important theorem, which made sensible the all schema of dynamic quantization:

Theorem . *The imposition of the second class constraints (4.1) does not change the form of Heisenberg equations, saving their classical form.*

Proof.

Let $|\mathcal{M}'\rangle, |\mathcal{N}'\rangle, \dots$ designate the basic vectors (3.3) constructed with the help of the regularized set of operators $\{a_N, a_N^+\}'$, and F' designate the Fock space with these basic vectors. The imposition of constraints (4.1) means that the space of physical states F is reduced to the regularized subspace $F' \subset F$. By definition for

any operator A in the regularized theory only the matrix elements $\langle \mathcal{M}' \,|\, A \,|\, \mathcal{N}' \rangle$ are considered and the operators a_N and a_N^+ with $|\, N \,| > N_0$ contained in the operator A are put equal to zero after the normal ordering. Therefore, in regularized theory the matrix elements of quantum DB for operators A and B corresponding to the constraints (4.1) are represented in the form

$$\langle \mathcal{M}' \,|\, [A,\, B]^* |\, \mathcal{N}' \rangle = \sum_{\mathcal{L}'} (\langle \mathcal{M}' \,|\, A \,|\, \mathcal{L}' \rangle \langle \mathcal{L}' \,|\, B \,|\, \mathcal{N}' \rangle - \langle \mathcal{M}' \,|\, B \,|\, \mathcal{L}' \rangle \langle \mathcal{L}' \,|\, A \,|\, \mathcal{N}' \rangle) \tag{4.2}$$

By definition of quantum DB the operators a_N and a_N^+ with $|\, N \,| > N_0$ contained in the operators A and B from (4.2) are normally ordered and then put equal to zero. In contrast to DB (4.2) in PB $[A,\, B\,]$ at the calculation of matrix elements $\langle \mathcal{M}' \,|\, [A,\, B\,] \,|\, \mathcal{N}' \rangle$ by the formula, similar to (4.2), the summation goes over all intermediate states (3.3). Suppose that the operator B is diagonal in basis (3.3) and does not depend on the operators a_N and a_N^+ with $|\, N \,| > N_0$. Then it is seen from (4.2) , that

$$\langle \mathcal{M}' \,|\, [A,\, B]^* \,|\, \mathcal{N}' \rangle = \langle \mathcal{M}' \,|\, [A,\, B\,] \,|\, \mathcal{N}' \rangle\ , \tag{4.3}$$

if in a right hand side of Eq.(4.3) the operators a_N , a_N^+ at $|\, N \,| > N_0$ in operator A are normally ordered and then are put equal to zero. The CR (3.6) imply that the Hamiltonian of the theory does not depend on the operators $a_N,\ a_N^+$. Therefore in (4.3) it is possible to substitute Hamiltonian H for operator B . It means that the theorem is true.

There is also classical variant of the Theorem.

The imposition of the second class constraints (4.1) does not change classical form of Hamilton equations for the remained degrees of freedom.

For provement we shall write out the formula for Dirac bracket in classic theory.

Let $\{\chi_\alpha\}$ and $\{\kappa_n\}$ designate the finite or infinite sets of the first and the second class constraints respectively. By definition this means, that

$$[\,\chi_\alpha,\, \chi_\beta\,] \approx 0, \tag{4.4}$$

$$[\,\chi_\alpha,\, \kappa_n\,] \approx 0 \tag{4.5}$$

$$[\,\kappa_m,\, \kappa_n\,] = c^{-1}_{mn} \tag{4.6}$$

Following to Dirac, we denote by the symbol $\approx$ the equalities modulo terms,which are proportional to constraints κ_n or χ_α . We shall pay attention that the matrix c^{-1}_{mn} in (4.6) is nondegenerate matrix which in general case is dependent on dynamical variables. The Hamiltonian of the system H is the first class quantity

$$[\,H,\, \chi_\alpha\,] \approx 0, \tag{4.7}$$

$$[\,H,\,\kappa_n\,] \approx 0 \tag{4.8}$$

In classic theory the Dirac bracket of any two quantities is defined by the formula

$$[\,\xi,\,\eta\,]^* = [\,\xi,\,\eta\,] - \sum_{m,n} [\,\xi,\,\kappa_m\,]\, c_{mn}\, [\,\kappa_n,\,\eta\,] \tag{4.9}$$

Evidently, for any quantities ξ and κ_n we have

$$[\,\xi,\,\kappa_n]^* = 0$$

From here it follows that the second class constraints κ_n can be put equal to zero before the calculation of Dirac brackets. Because of (4.8) and (4.9) the equation

$$[\,\xi,\,H\,] \approx [\,\xi,\,H]^* \tag{4.10}$$

is valid. The weak Eq.(4.10) means, that the equations of motion obtained with the help of the Poisson brackets and Dirac brackets, essentially coincide.

In the context of our method according to (3.6) the weak equalities (4.5) and (4.8) are transformed in strong equalities. That is why from (4.9) it immediately follows, that $[\,\xi,\,H\,] = [\,\xi,\,H]^*$ for any quantity ξ . It means that the theorem is true.

Consequence. *The regularized theory is general covariant.*

This Consequence directly follows from the proved theorem. Indeed, the equations of motion in regularized theory, coincide in the form with classical one which are general covariant.

We call our quantization method as dynamic namely for the reason, that in this method the regularization is ideally agreed with dynamics of the system. Once again we pay attention to the fact, that the regularization does not touch the gauge group of the theory i.e. the group $\mathcal{G}$ in regularized theory remains such, which it is in classical theory.

Now we shall state more formal approach to the dynamic quantization method. This approach, being perhaps less natural, is more logically harmonic and also permit to simplify some calculations.

The basis of such approach is

Assumption. *The theory is assumed regularized so that the following axioms are true:*

Axiom R1. *All states of the theory, having the physical sense, are obtained from the ground state* $|\,0\,\rangle$ *with the help of the creation operators* a_N^+ with

$|\,N\,| < N_0$:

$$|\, n_1,\, N_1; \ldots ; \, n_s,\, N_s \,\rangle = (n_1! \cdot \ldots \cdot n_s!\,)^{-\frac{1}{2}} \cdot (a^+_{N_1})^{n_1} \cdot \ldots \cdot (a^+_{N_s})^{n_s} \,|\, 0 \,\rangle \ ,$$

$$a_N \,|\, 0 \,\rangle = 0 \tag{4.11}$$

The states (4.11) form orthonormal basis of the physical states space F' *of the theory.*

Axiom R2. *The states (4.11) satisfy the conditions*

$$\chi_{ab}(x)\,|\ \rangle = 0, \qquad \phi_a(x)\,|\ \rangle = 0 \tag{4.12}$$

Axiom R3. *The dynamic variables* $e^a_i(x)$ *transfer the state (4.11) into some superposition of states of the theory. This superposition contains the* **all** *states for which one of occupation number differs per unit from corresponding occupation number of the state (4.11) and the rest occupation numbers coincide with corresponding occupation numbers of the state (4.11).*

Axiom R4. *The equations of motion and constraints for physical fields* $e^a_i(x), \omega^{ab}_i(x)$ *coincide in form (up to arrangement of the operators) with the corresponding classical equations of motions and constraints .*

The Axioms R1 - R3 are analogous to Axioms 1- 3 in the nonregularized theory. The Axiom R4 replaces the Theorem. It postulates a correct form of equations of motion and constraints in agreement with classical mechanics.

Finally let us discuss briefly a serious problem of operators ordering in regularized Heisenberg equations in general form. The fields $e^a_i(x)$ and $\omega^{ab}_i(x)$ will be designated by one symbol $\Phi(x)$ and the fields $e^a_0(x)$ and $\omega^{ab}_0(x)$ by the symbol $\lambda(x)$. The problem of operators ordering arises when we find out the selfconsistency of the theory [3]. Let us write out the Heisenberg equation in general form:

$$\dot{\Phi}(x) = F(\lambda,\, \Phi\,)(x) \tag{4.13}$$

Here the field $\lambda(x)$ determines the Hamiltonian according to (2.3). In (4.13) F is a local function of the fields $\lambda(x)$ and $\Phi(x)$ and linearly depends on the field λ . By definition the field λ is placed at the left of the fields Φ in function $F(\lambda, \Phi\,)$. According to (4.13) we have for any fields λ_1 and λ_2 :

$$\delta_i \Phi = \delta t_i \, F(\lambda_i,\, \Phi\,)\,, \qquad i = 1, 2$$

Consider the quantity $(\delta_1\delta_2 - \delta_2\delta_1\,)\,\Phi$ which is denoted by $\delta_{[1\,2]}\Phi$. The necessary condition of the selfconsistency of the theory is the possibility of a such operators ordering in Eq.(4.13) that the weak equality

$$\delta_{[1\,2]}\Phi \approx \delta t_1\,\delta t_2\,F\,(\lambda_{[1\,2]},\,\Phi\,) \tag{4.14}$$

takes place. Here the field $\lambda_{[1\,2]}$ is a bilinear antisymmetric form of the fields λ_1 and λ_2 and it generally depends on the field Φ. The first class constraints χ_{ab} and ϕ_c , which are vector fields, on the group $\mathcal{G}$, do not contain the creation and annihilation operators $a^+_N,\ a_N$. The last property obviously is used in calculations in the next Section.

5. Perturbation theory.

In this Section we shall show, in what way the field coefficients at operators a_N and a^+_N in expansions of the tetrad and connection fields (see (3.12) can be find step by step. The calculations begin without quantum corrections (loops). Then the result is corrected with allowance for quantum fluctuations. This proves to be formally equivalent to the expansion in the number N_0 . As shown below, the formal expansion in the number N_0 is equivalent to expansion in dimensionless parameter $(\,\Lambda\,\kappa\,)^2$, where Λ is the cut off momentum of the theory. If the cut off momentum much less of Planckian momentum, then $(\,\Lambda\,\kappa\,)^2 \ll 1$.

The description in this Section is very sketchy. The detailed study of PT and investigation of concrete problems must be carried out with the help of the dynamic method in the special woks.

Calculations are begin with zero approximation $e^{a(0)}_i(x)$ and $\omega^{ab(0)}_i(x)$. The tetrad and connection fields in zero approximation satisfy equations and constraints (2.8), (2.9) and (2.11) and they do not depend on creation and annihilation operators. However, in the zero approximation tetrad and connection fields are operators on the group of gauge transformations $\mathcal{G}$ according to (3.1), (3.16), (3.17). Thus, the fields $e^{a(0)}_i(x)$ and $\omega^{ab(0)}_i(x)$ satisfy equations of motion (2.8) and (2.10), and the constraints $\chi^{(0)}_{ab}$ and $\phi^{(0)}_c$ composed from these fields, by definition annulate the state $|\,0\,\rangle$.

In the first approximation the all quantum states (4.11) from regularized space are involved in consideration. The tetrad and connection fields are expanded in the first approximation as follows (see (3.9) and (3.12)):

$$e^a_i(x) = \frac{1}{\sqrt{2}}\sum_{|N|<N_0}(\,a_N e^a_{Ni}(x) + a^+_N \bar{e}^a_{Ni}(x)\,) + e^{a(0)}_i(x) \equiv e^{a(0)}_i(x) + e^{a(1)}_i(x),$$

$$\omega_i^{ab}(x) = \frac{1}{\sqrt{2}} \sum_{|N|<N_0} (\, a_N\, \omega_{Ni}^{ab}(x) + a_N^{+}\, \bar{\omega}_{Ni}^{ab}(x)\,) + \omega_i^{ab(0)}(x) \equiv$$

$$\equiv \omega_i^{ab(0)}(x) + \omega_i^{ab(1)}(x) \tag{5.1}$$

We remark, that at our approach the field e_0^a and ω_0^{ab} playing the role of the Lagrange multipliers remain numerical:

$$e_0^{a(s)} = 0, \qquad \omega_0^{ab(s)} = 0, \qquad s = 1, 2, \ldots \tag{5.2}$$

Substituting the fields from (5.1) in equations (2.8), (2.9) and (2.11) and taking into account the equation (5.2) and that fact, that the fields $e_i^{a(0)}$, $\omega_i^{ab(0)}$ satisfy all classical equations, we obtain:

$$\varepsilon_{abcd}\varepsilon_{ijk}\,\{\, R_{ij}^{ab(0)} e_k^{c(1)} + 2\, e_k^{c(0)} \nabla_i^{(0)} \omega_j^{ab(1)}\,\} = 0,$$

$$\varepsilon_{ijk}\,(\, \nabla_i^{(0)} e_j^{a(1)} - e_{bi}^{(0)} \omega_j^{ab(1)}\,) = 0, \tag{5.3}$$

$$\varepsilon_{abcd}\varepsilon_{ijk}\,\{\, R_{0i}^{ab(0)} e_j^{c(1)} + e_j^{c(0)} \nabla_0\, \omega_i^{ab(1)} + e_0^c \nabla_i^{(0)} \omega_j^{ab(1)}\,\} = 0,$$

$$\nabla_0\, e_i^{a(1)} - e_{b0} \omega_i^{ab(1)} = 0 \tag{5.4}$$

Equations (5.3) are the constraints, and equations (5.4) are equations of motion for the tetrad and connection fields. Taking the matrix elements of Eqs. (5.3) and (5.4) of the form $\langle\, 0\, | \ldots |\, N\, \rangle$, we find

$$\varepsilon_{abcd}\varepsilon_{ijk}\,\{\, R_{ij}^{ab(0)} e_{Nk}^{c} + 2\, e_k^{c(0)} \nabla_i^{(0)} \omega_{Nj}^{ab}\,\} = 0, \tag{5.5a}$$

$$\varepsilon_{ijk}\,(\, \nabla_i^{(0)} e_{Nj}^{a} - e_{bi}^{(0)} \omega_{Nj}^{ab}\,) = 0, \tag{5.5b}$$

$$\varepsilon_{abcd}\varepsilon_{ijk}\,\{\, R_{0i}^{ab(0)} e_{Nj}^{c} + e_j^{c(0)} \nabla_0\, \omega_{Ni}^{ab} + e_0^c \nabla_i^{(0)} \omega_{Nj}^{ab}\,\} = 0, \tag{5.6a}$$

$$\nabla_0\, e_{Ni}^{a} - e_{b0}\, \omega_{Ni}^{ab} = 0 \tag{5.6b}$$

Thus, the equations of constraints (5.5) and motions (5.6) break up on separate equations with given number N . The same property takes place for fields $e^a_{N_1 \ldots N_s\, i}$ and etc., if quantum fluctuations do not taken into account. For example, we have the following analog of equations (5.5 b) and (5.6 b) for the fields $e^a_{N_1 N_2\, i}$ and $\omega^{ab}_{N_1 N_2\, i}$:

$$\varepsilon_{ijk}\,\{\, \nabla_i^{(0)} e^a_{N_1 N_2\, j} + e_{bj}^{(0)} \omega^{ab}_{N_1 N_2\, i} + \omega^{ab}_{N_1\, i} e_{N_2\, bj} + \omega^{ab}_{N_2\, i} e_{N_1\, bj}\,\} = 0\,, \tag{5.7}$$

$$\nabla_0 e^a_{N_1 N_2\, i} - e_{b0} \omega^{ab}_{N_1 N_2\, i} = 0 \tag{5.8}$$

It is seen , that in opposite to Eqs.(5.5) and (5.6) , which are uniform relative to the fields $e^a_{N\, i}$ and ω^{ab}_{Ni} , Eqs.(5.7) are not uniform relative to the fields $e^a_{N_1 N_2\, i}$ and $\omega^{ab}_{N_1 N_2\, i}$.

To solve Eqs. (5.5) - (5.8) it may be applied the following scheme. At first it is necessary to solve the uniform system of Eqs. (5.5) and (5.6) relative to the

fields e^a_{Ni} and ω^{ab}_{Ni} . Then we solve the linear nonuniform system of equations including Eqs. (5.7) and (5.8) relative to the fields $e^a_{N_1 N_2\, i}$ and $\omega^{ab}_{N_1 N_2\, i}$. This system of equations is dependent on the fields e^a_{Ni} and ω^{ab}_{Ni} found before. Further this process spreads on the higher fields $e^a_{N_1 \dots N_s\, i}$ and etc. Without the quantum fluctuations the arising equations relative to the fields $e^a_{N_1 \dots N_s\, i}$ and etc. are linear nonunoform equations dependent on the fields found on previous steps.

The number of constraints and equations (5.5) and (5.6) is equal to fourty, and number of the unknown functions e^a_{Ni} and ω^{ab}_{Ni} (at fixed index N) is equal to thirty. Nevertheless the system of Eqs.(5.5) and (5.6) has nonzero solutions. Really, the equations of motion (5.6) have the solutions at any meanings of fields e^a_0 and ω^{ab}_0 . This is obvious for initial Eqs. (2.8), (2.9) and (2.11). Therefore the obtained from Eqs. (2.8), (2.9) and (2.11) the equations (5.5) and (5.6) relative to the fields e^a_{Ni} and ω^{ab}_{Ni} have solutions.

Solving Eqs.(5.5) and (5.6) one must use the initial conditions (3.11) for the fields e^a_{Ni} at $t = t_0$. The field ω^{ab}_{Ni} is expressed unambiguously through the fields e^a_{Ni} and $\dot{e}^a_{Ni}$ with the help of Eqs. (5.5 b) and (5.6 b). Then equations (5.5 a) and (5.6 a) result in linear differential equations of the second order relative to the fields e^a_{Ni} .

From normalization condition (3.13) and equations (5.5) and (5.6) it is seen, that the fields e^a_{Ni} and ω^{ab}_{Ni} are proportional to the gravitational constant κ . Now become clear the sense of multiplication on constant κ^2 in the right hand side of normalization condition (3.13) . The expansion (5.1) and proportionality of the fields e^a_{Ni} and ω^{ab}_{Ni} to the constant κ gives the possibility of fulfilment of the Poisson brackets (2.13) in lowest approximation relative to the creation and annihilation operators (in nonregularized theory).

Thus, the fields e^a_{Ni} and ω^{ab}_{Ni} are found according to the following rule. One must find the fields e^a_{Ni} and ω^{ab}_{Ni} , constraind to the conditions (3.11) at $t = t_0$, (3.13) and equations (5.5) and (5.6). The tetrad and connection fields composed from found fields according to (5.1) (where summation goes over all N), must satisfy the Poisson brackets (2.13) in lowest approximation. Then from the all set of the fields e^a_{Ni} and ω^{ab}_{Ni} we choose the regularized subset $\{ e^a_{Ni}, \omega^{ab}_{Ni} \}'$ with $|N| < N_0$.

We notice, that according to equations (5.7) we have

$$e^a_{N_1 N_2\, i} \sim \kappa^2 \, , \qquad \omega^{ab}_{N_1 N_2\, i} \sim \kappa^2 \tag{5.9}$$

Consider shortly the question about quantum fluctuations or loops.

Denote by $e^{a(s)}_{Ni}$, $s = 0, 1, \dots$, the s-loop contributions in the fields e^a_{Ni} . Thus the considerred above fields e^a_{Ni} correspond to the fields $e^{a(0)}_{Ni}$ in new notations.

By the same way , as equations (5.5) - (5.8) were obtained, we find the following equations in one-loop approximation:

$$\varepsilon_{ijk}\,\{\,\partial_i e^{a(1)}_{Nj} + \sum_{|M|<N_0} [\,2\,(\,\omega^{ab(0)}_{NMi}\,\bar{e}^{(0)}_{Mbj} + \bar{\omega}^{ab(0)}_{Mi}\;e^{(0)}_{NMbj}\,)+$$

$$+(\,\omega^{ab(0)}_{M;Ni}\,e^{(0)}_{Mbj} + \omega^{ab(0)}_{Mi}\;e^{(00)}_{M;Nbj}\,)\,]\,\} = 0 \qquad (5.10)$$

From dimensional considerations (see (5.9)) it is easy to understand, that the sum in last equation is of the order $(\,\Lambda\,\kappa\,)^2$, where Λ is the cut-off momentum of the theory. The expansion at successive accounting of quantum fluctuations goes namely in dimensionless parameter $(\Lambda\kappa\,)^2$. Now the question about conception "small number" of the physical degrees of freedom is clearified. This is such a number, at which

$$(\Lambda\kappa)^2 \ll 1 \qquad (5.11)$$

Condition (5.11) means, that the cut-off momentum is much less, than Planckian momentum. In this case the account of quantum fluctuations can be made with the help of finite PT so , as shown above.

6. Conclusion

Thus, we described the new scheme of canonical quantization of gravity. The constucted quantum theory has following basic properties.

A. If x-space is compact than the number of the physical degrees of freedom is finite.

B. The Heisenberg equations for tetrad and connection and other fields are of classical form (up operators ordering).

C. The constucted theory is general covariant.

Unfortunately, we have to do an essential reservation. The mathematical correctness of the theory will be completely established only, when the problem of operators ordering in equations of motions will be solved. In this paper the mathematical correctness is established for decided here problems of ultraviolet divergences and general covariance of the theory. On the first sight the problem of creation and annihilation operators ordering in Section 5 is resolved automatically, since the coefficients at the creation and annihilation operators in equations of motion and constraints are put equal to zero. Thus we obtain the equations for the fields $\Phi_{\mathcal{N}}$. However, in this case the question about correctness of these equations arises. This question needs special examination.

Pay our attention, that the condition (5.11) of existence of PT is not necessary for the mathematical correctness of the theory. In our opinion, the physically sensible quantum theory of gravity should not be restricted by condition (5.11). Nevertheless, it is possible, that in some concrete examples the condition (5.11) will effectively take place.

Though in this paper the theory was built in the case of pure gravity, the inclusion of matter in the theory on the first sight cannot lead to principle difficulties under dynamical quantization. In this direction the most interest for us presents the studies of supergravity, since the property of supersymmetry of the theory is easier established on the equations of motion playing the main role in dynamic quantization method.

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